Open pit design and production scheduling deals with the quest for the most profitable mining sequence over the mine's life. The dynamics of mining ore and waste and interactions with spatial grade uncertainty make the prediction of the optimum mining sequence a challenging task. This paper examines an optimization approach to open pit production scheduling based on the effective management of waste mining to maximize net present value (NPV) and in relation to the presence of grade uncertainty for both ore and waste. The approach considers an economic model, the specific mine set-up, mining and processing specifics including production equipment, as well as the development of a combinatorial optimization formulation that integrates multiple grade realizations of the deposit to produce a single optimal life-of-mine production schedule. The efficient use of grade uncertainty and mining rates leads to schedules that are risk-resilient, as well as a substantial improvement in project NPV compared to conventional methods. A case study from a gold mine demonstrates the approach and illustrates the potential economic benefit.
INTRODUCTION

In surface mining, the need to assess and manage risk for project valuation and decision-making translates to the need to assess and manage risk in any pertinent parameter of open pit design and production scheduling. This can only be achieved if quantified uncertainty is taken into account in the mine optimization process. Geological uncertainty is seen as a major contributor to not meeting project expectations. The problem of quantifying geological uncertainty in open pit design and production scheduling can be addressed in the context of stochastic simulation (Dimitrakopoulos, 2004). Optimization in mine planning has been accepted as a set of techniques that introduce analytical mathematical methods into mine planning (Lane, 1999). In the presence of risk, effective optimization calls for the use of advanced mine optimization techniques that are able to take into account the stochastic nature of several influencing variables and constraints. Unfortunately, to date there has been limited success in developing such techniques, which remain the subject of ongoing research (e.g. Dimitrakopoulos and Ramazan, 2004; Ramazan and Dimitrakopoulos, 2004; Menabde et al., 2004).

This paper presents an optimization approach that integrates grade uncertainty into the optimization of long-term production scheduling. A general framework for long-term production scheduling is reviewed and extended through combinatorial optimization to allow effective minimization of risk in not achieving production targets due to geological uncertainty. The approach has the ability to minimize deviations from production target variables to acceptable ranges. An application developed in a large open pit gold mine is presented to show the potential economic benefits of the propose approach. Some of the production scheduling concepts considered in the approach proposed herein originate from Russian mining (Rzhenevsky, 1968) and are considered in Tan and Ramani (1992) in formulating optimization models. Recently, Godoy (2003) revisits the concepts in the context of modern open pit scheduling optimization and, in particular, scheduling optimization based on nested pits (Whittle and Rozman, 1991). The framework considers the open pit production scheduling optimization process as the determination of a sequence of depletion schedules involving the removal of at least two types of material, namely, ore and waste. Two major technical constra-
A feasible domain of ore production and waste removal, bounded by the worst case (top) and best case (bottom) schedule.

The key variable involved in the determination of an optimal waste removal curve is the mining capacity. Tan and Ramani (1992) proposed a linear programming (LP) model to solve such optimization problem. This LP model is used herein, extended to include periodic stabilisation of mining rates, so as to avoid solutions with unpractical fluctuations in mining capacity, and metal optimization. The optimization model delivers a life of mine schedule of waste removal and a prescription for the formation of mining capacity, given a predefined ore demand function and a set of possible models of mining equipment. This schedule maximizes the project's NPV for a set of economic and technological parameters. However, the formulation similarly to that in Tan and Ramani (1992), does not provide the physical mining sequence and, therefore, does not provide a complete solution to the long-term scheduling problem.

To overcome this limitation, a procedure is proposed herein that consists of decomposing the long-term scheduling problem into two sub-problems. First, the determination of optimum mining rates for the life-of-mine; and, second, the generation of a detailed mining sequence constrained by the previously determined mining rates. The approach is general and independent of the scheduling formulation used to produce the detailed mining sequence. The first sub-problem deals with the objectives of ore production, stripping ratios, major investment in equipment purchase and average operational costs. The second sub-problem focuses
on the spatial evolution of the mining sequence and the equipment use and provides a more precise assessment of operational costs.

In the following sections, the proposed multi-stage approach is first presented, emphasizing the combinatorial optimization part of the approach that generates the risk-based life of mine production schedule. Then, an application at an open pit gold mine elucidates the practical aspects of the approach and provides a comparison with the traditional optimization approach.

**A NEW RISK-BASED APPROACH TO PRODUCTION SCHEDULING**

The risk-based (stochastic) approach to production scheduling optimization proposed here is conceptually very different from the traditional deterministic one. In all deterministic approaches the optimization formulation processes a single estimated orebody model to produce a mining schedule. Since this type of estimated orebody models is based on imperfect geological knowledge, estimation errors, including the inevitable smoothness of "average" mining block grade estimates, are propagated to the various mining processes involved in the optimization (in general, a non-linear operation). The final result is a single and often biased, forecast for the economic outcome of the production schedule (Dimitrakopoulos et al., 2002). In the stochastic framework, geological uncertainties are characterised by a series of equally probable models of the orebody, as produced by conditional simulation techniques (Dimitrakopoulos, 1998; 2004). The multi-stage optimization algorithm presented herein takes simultaneously all these models into account, so as to produce the single optimal mining schedule under grade uncertainty. A consequence of the above is that instead of providing a single biased forecast for the economic outcome, this multistage approach yields a range of possible economic outcomes along with a single risk-based schedule. One of the most important features of the approach is its ability to minimize the ranges of variation of economic outcomes, thus allowing for the minimization of the geological risk associated with the generated schedule.

The proposed approach first generates a series of mining schedules, each corresponding to a simulated realisation of the spatial distribution of grades. These mining sequences are optimized within a common feasible domain and post processed to provide a single optimal mining sequence, which minimizes the chance of deviating from the target production figures. The approach proceeds in the following stages:

1. Derive a stable solution domain of ore production and waste removal stable to all simulated models of the distribution of grades within the orebody.
2. Determine the optimum production schedule within the solution domain derived in the first stage above, using a linear programming (LP) formulation. This will generate optimum mining rates for the life-of-mine scheduling.
3. For each one of the simulated models, generate a physical mining sequence constrained to the mining rates derived in the second stage (note that all are sub-optimal mining schedules).
4. Combine, using combinatorial optimization, the mining sequences generated in the third stage to produce a single optimal mining sequence that minimizes the chances of deviating from production targets.

**First stage: Derivation of the stable solution domain**

Start by considering N equi-probable simulated orebody models $S_1, ..., S_n$, mapping the space of uncertainty for a given continuous attribute; an ultimate pit limit; and a sequence of cutbacks. Generate a series of N cumulative graphs of ore production and waste removal (feasible solution domain), one for each simulated orebody model. The "stable solution domain" (SSD) is then defined as the intersection of all feasible domains (Godoy and Dimitrakopoulos, 2004). This solution represents a domain that provides 100% confidence on the contained reserves. The procedure is general and independent of the objectives driving the optimization of the production scheduling. Figure 3 illustrates the SSD for a series of simulated models in a gold open pit mine.
Second stage: Schedule optimization

The second stage corresponds to the optimization of the production schedule in terms of ore production and waste removal. It incorporates the LP optimization model discussed in the previous section. Note that an additional main difference here from Tan and Ramani (1992) is that the solution domain is now based on a series of simulated orebody models. Involving also the economic parameter unit purchase and ownership costs of each type and model of mine equipment available, the stabilization of the mining rate over time periods is determined as a search for the balance between the purchase and ownership costs of the production capacity. This represents a direct incorporation of the capital investments in the production scheduling optimization. Further details on the optimization model in this second stage are in Godoy and Dimitrakopoulos (2004).

Third stage: Mining sequencing

This third stage aims to produce a series of physical schedules describing the detailed spatial evolution of the working zones over the life-of-mine. Any formulation able to perform mining sequencing can be used for this task. The requirement is the ability to constrain the sequencing to obey slope constrains, maximize the equipment utilization, and meet mill requirements while matching the mining rates previously derived by the optimization. The procedure consists of producing multiple mining sequences, one for each simulated grade model. These multiple alternative mining sequences present two specific properties that will allow the derivation of a single sequence: (a) They are all technically feasible solutions that maximize the project’s NPV within a common stable solution domain. (b) They are based on distinct but equiprobable models of the spatial distribution of grades within the mineral deposit.

It is important to stress at this point that selecting and using one or few “representative” simulated realizations of the orebody for scheduling proposes is an erroneous practice and leads to misleading schedules.

Fourth stage: Combinatorial optimization

The fourth stage requires a combinatorial optimization algorithm to be developed to generate a single mining sequence from the alternative sequences produced in the third stage. The algorithm that has been developed is based on simulated annealing, a technique for solving combinatorial optimization problems such as the minimization of functions of many variables (Kirkpatrick et al., 1983). The key idea is to continuously perturb a sub-optimal configuration until it matches some pre-defined characteristics expressed in an objective function at an acceptable level.

The optimization starts by selecting an initial mining sequence where blocks with maximum probability of belonging to a given period are frozen for that period. The maximum probability threshold is user defined. Subsequently, the selected initial sequence is
perturbed by randomly swapping selected blocks between candidate periods. All favourable perturbations (i.e. where the objective function is lowered) are accepted, whilst all unfavourable perturbations are accepted with an exponential probability distribution. The optimization is considered complete when additional perturbations do not lower the value of the objective function or when a specified minimum objective function value is reached.

The objective function is defined as a measure of the difference between the desired characteristics and those of a candidate mining sequence. In this case, the measure is the average deviation from the production targets for a given mining sequence over a series of S simulated grade models. It is defined as the sum of N components:

$$O = \sum_{n=1}^{N} \left[ \sum_{s=1}^{S} \left( \theta_n(s) - \bar{\theta}_n(s) \right) + \sum_{s=1}^{S} \left( \omega_n(s) - \bar{\omega}_n(s) \right) \right]$$  \hspace{1cm} (1)

where, n=1,...,N are the component objective functions, each corresponding to one of the production schedule periods. For each n component, the objective function measures the average deviation of ore and waste production $\theta_n(s)$ and $\omega_n(s)$ of the perturbed mining sequence from the target productions $\bar{\theta}_n(s)$ and $\bar{\omega}_n(s)$ over all S simulated grade models with s=1,...,S. The decision of whether to accept or reject a perturbation is based on the change to the objective function in Equation 1.

Recalculations of the global objective function can be replaced by a selective update of the component objective functions involved in the perturbation. The resulting sequence meets the production target for each period with minimum chance of deviation, i.e., this mining sequence will achieve the production targets, within the prescribed mining rates, given any of the simulated orebody models. The swapping mechanism is an important aspect of the annealing procedure above. To guarantee the feasible final solution, the perturbation mechanism must be able to recognize the spatial evolution of the mining sequence. To accomplish this, the swapping mechanism is set to limit the candidate periods for any given block to only those that will have physical access to the block without violating slope constraints.

In addition to the objective function and the perturbation mechanism, a critical aspect of simulated annealing-based algorithms is a prescription for when to accept or reject a given perturbation. The acceptance probability distribution is given by the Metropolis criterion (Metropolis et al, 1953):

$$P\{\text{accept}\} = \begin{cases} 1, & \text{if } O_{\text{new}} \leq O_{\text{old}} \\ e^{-\frac{O_{\text{old}} - O_{\text{new}}}{T}}, & \text{otherwise} \end{cases}$$

All favorable perturbations updated from $O_{\text{old}}$ to $O_{\text{new}}$ ($O_{\text{new}} \leq O_{\text{old}}$) are accepted and unfavorable permutations are accepted with an exponential probability distribution. The algorithm is stopped when a low objective function value ($O_{\text{min}}$), indicating convergence, is reached. A so-called “cooling” function controls the rate of decrease in time of the parameter T, also known as “temperature”, of the exponential distribution. The higher T is, the greater the probability that an unfavorable perturbation will be accepted.

The parameter T must not be lowered too fast because the mining sequence may get trapped in a sub-optimal situation and never converge. However, if T is lowered too slowly, the convergence may be unnecessarily slow. The specification of how to lower T is known as the annealing schedule. The idea is to start with an initially high $t_0$ and lower it by a multiplication factor $\lambda$ whenever enough perturbations have been accepted ($K_{\text{accept}}$) or too many have been tried ($K_{\text{max}}$). The algorithm is stopped if $K_{\text{max}}$ is reached $S_{\text{tp}}$ times, where $S_{\text{tp}}$ is the parameter known as “stopping number”. The algorithm is also stopped if a maximum number of swaps is reached or after reaching a maximum

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<th>Table N°1: Parameters used for the annealing schedule.</th>
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number of swaps with no change in the objective function. These parameters are named MaxSwap and MaxNo-Change respectively (see Table 1).

The method presented in this section provides a framework for the derivation of a single schedule that minimizes the chances of deviating from production targets, given the uncertainty assessment from the available information. Precedence constraints built into the perturbation mechanism are designed to recognize the spatial evolution of the mining sequence, which is restricted by pit slope constraints. These mining sequences are produced by an external mining sequence algorithm and must reflect mining practices and technological constraints.

**APPLICATION IN AN OPEN PIT GOLD MINE**

An application of the proposed method is carried out at the Fimiston open pit mine, Western Australia, Australia’s premier gold mine. This application starts by the development of a “Base Case” schedule for the life of the mine. The development of a Base Case schedule aims to produce a benchmark against which the potential economic benefits of the new risk-based optimization approach can be evaluated.

The Base Case schedule was developed using a traditional estimated model for the distribution of grades, as used in the conventional approach. The mining capacity was formed with a combination of Komatsu PC8000 face shovels, CAT994 loaders and CAT793C
trucks. A constant mining capacity of 85Mtpa was adopted. The schedule of ore production was identified with the mill demand. Both the schedule of mining capacity and ore production are presented in Figure 4. It is important to note that the fluctuations in ore production do not indicate a variable mill production rate. The mill production rate is constant over the life of the mine. Periods characterised by a reduction in ore demand only indicate input of ore from external sources such as, for example, underground operations and stockpiles. A risk analysis on the Base Case schedule, using a set of simulated models, was also carried out. This risk analysis was developed by taking the Base Case mining sequence, which indicates which blocks are to be mined in each period, and evaluating the schedule outcome for each one of the simulated models. The procedure generates a distribution of responses, or a range of alternative outcomes for key project indicators and is similar to that employed in Dimitrakopoulos et al. (2002).

Figure 5 presents the Base Case predictions and the risk profile obtained for the annual and cumulative discounted cash flows, respectively. The expected NPV is approximately 11% less than the indicated by the initial predictions of the Base Case schedule. Figure 6 presents the results obtained from the risk analysis on the ore production and the initial predictions of the Base Case schedule. The average expected deviation from the Base Case prediction shows a deficit of approximately 1.3Mt per year. This result shows that the Base Case schedule is unable to meet the predicted mill feed tonnage.

Note that the use of optimal mining rates, using the approach in the second stage without grade risk and in combination with the conventional methods used in the Base Case, provides some relatively small improvement and leads to improvements of total NPV. However, similarly to the Base Case schedule, it does not meet production targets, as shown in Godoy (2003).

The application of the risk-based approach starts, as outlined previously, by the derivation of the stable solution domain, the optimization in stage two carried out within the SSD. The resulting prescription of ore and waste production and the selection of mining equipment forming the required mining capacity are used to generate the mining sequences, one sequence for each simulated model. Last, the simulated annealing procedure was used to combine these multiple mining sequences. Figure 7 shows the component objective functions versus the number of attempted perturbations in the present application. The optimization stopped after 202,669 perturbations, with 8716 being accepted, as it reached the maximum number of attempts with no change in the objective function.

The exceptional performance and effectiveness of the proposed method and the effects of managing risk is further demonstrated in the results shown in Figure 8. The figure shows the final schedule and the risk profile.
Figure N°7

Component Objective Functions versus Attempted Swaps

Simulated annealing: Component objective function values versus attempted number of swaps.

obtained from the risk analysis in ore production. The bars indicate the absolute average deviation from the target. The largest deviations belonging to years 2002, 2005 and 2008 are respectively 357,000t, 347,000t and 265,000t. The magnitude of these deviations is considered very small and could be easily managed by rehandling ore from alternative sources, especially for those years presenting a shortfall.

In terms of NPV, shown in Figure 9, the expected outcome corresponds to an increase of 28.3% in relation to predicted NPV for the Base Case schedule. This difference reflects both the deferment of waste mining and the reduction in the life of the mine. One of the major contributions to the increased NPV comes from the recovered metal, the risk-based schedule recovers the same metal quantity first predicted by the Base Case but a main difference is that it does so sooner.

Important aspect of the case study is that it demonstrates how risk-inclusion leads to a counterintuitive risk reduction and simultaneous increase of NPV, that are both substantial. In addition, the case study makes a distinct case for risk-based (stochastic) optimization against what is seen as the inherent limits of conventional technologies.

CONCLUSIONS

This paper presented a new risk-based optimization formulation for long-term production scheduling. This new multi-stage formulation was used to produce a minimum-risk and higher profitability life-of-mine schedule for a large open pit gold mine. The results obtained document that the proposed optimization approach has the potential to considerably improve the economic state and forecasts for the life-of-mine, when compared with conventional (non-risk based) scheduling practices. The results not only show a potential increase of 28.3% in the value of the mine, but also provided a schedule that minimizes the chance of deviating from mill feed requirements.

Major contributions to the increased NPV come from: (a) the recovered metal, the risk-based schedule recovers the same metal quantity first predicted by the traditionally generated Base Case but a main difference is that it does so sooner; and (b) the simultaneous use of simulated orebodies allows for the effective management of the upside potential and downside risk from the deposit's grade and metal uncertainty.

The approach has shown to be able to capitalise in both the deferment of waste mining and the assessment as well as inclusion of grade uncertainty, leading to maximized economic returns and driving the mining sequence through zones where the risk of not achieving the target ore production is minimized. The approach provides not only a risk resilient solution to the production scheduling problem but an increase in asset value by considering an inherent source of uncertainty and risk. This ability represents a...
major advance in the risk management of open pit mining and makes an eloquent case for the need to implement and further develop risk-based optimization approaches as an alternative framework to conventional optimization.

Lastly, one should note once again the counter intuitive observation from the case study presented. Unlike the “common sense” link of high risk to high rewards, the case study presented above demonstrates how risk-inclusion can lead to both risk reduction and simultaneous increase of NPV, both of which can be substantial. This is one of the several reasons in support for the further research, development and utilisation of risk-based optimization, as well as the move forward from the current conventional industry practices, concepts and their limits.

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